Higher-Order Risk Premium, Stock Return Predictability, and Rare Event Dynamics

By

Zhenzhen Fan, Xiao Xiao, Hao Zhou

Nancy R. Xu

Boston College

July 12, 2019, CICF

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- "Rare Event Dynamics" Higher-order risk premium could suggest some underlying behaviors of rare events in the market

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$$g(r,T) = \frac{1}{T-t}\sum_{i=1}^{n} g(r_i),$$

$$g(r) = 2(\exp(r) - 1 - r)$$

$$\approx \underbrace{r^2}_{\text{pure second moment}} + \underbrace{\frac{1}{3}r^3 + \frac{1}{12}r^4 + \ldots}_{\text{higher-order moments}}$$

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- The total varaince risk premium is $VRP_t = E_t^Q[g(r(t,T))] - E_t^P[g(r(t,T))]$
- The pure second moment: $PVRP_t = E_t^Q\left[\frac{1}{T-t}r(t,T)^2\right] - E_t^P\left[\frac{1}{T-t}r(t,T)^2\right]$
- The higher-order moments: $HRP_t = VRP_t - PVRP_t$

The Main Finding: (2) Empirical Properties

- S&P 500 index and its option data from 1996 to 2016
- PVRP is on average positive. In contrast, the average HRP, mainly attributed to the risk premium related to the third moment of returns, has the opposite sign
- HRP reflects compensation for unexpected large and discontinuous movement of stock returns, while PVRP represents compensation for uncertain continuous and diffusive movement of stock return variance.

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- Exercise 2: asset allocation
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- Exercise 3: predicting momentum returns
  HRP is the main contributor to momentum return predictability, in an up-to-date horse race.

The Main Finding: (3) Economic Interpretations

- Goal: understand the mechanism of the predictability of PVRP and HRP together in a consumption-based framework
- Revised model: Bollerslev, Tauchen and Zhou (2009) + Wachter (2013). The key change in the assumption is the jump shock in the consumption growth process, which now generates the higher-order (skewness) risk premium.
- The model is able to match the predictability signs of both risk premiums.

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Interesting paper, examining and pushing the big agenda of understanding risk compensations associated with higher moments in empirical asset pricing.

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1. Interpretations of negative spikes in the RP measures
2. Asset allocation exercise

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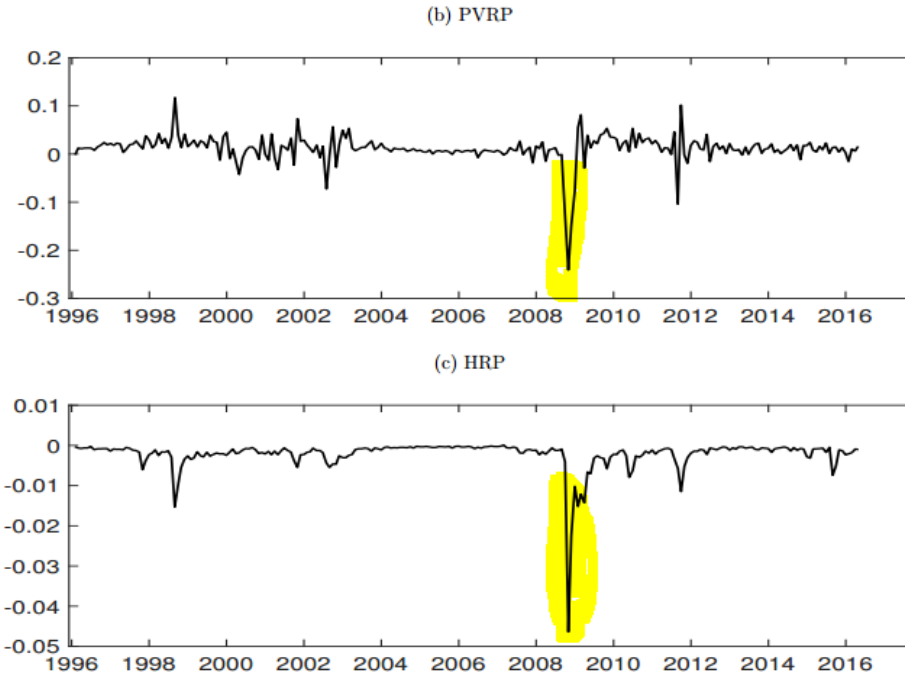
- Could the large realized variances (as proxy for the physical leg) result in the major spikes in VRP and PVRP? $\Rightarrow$ Concern of mismeasurement



correlation=0.39

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- For variance, variance sellers makes money when investors face economic turmoil and thus are willing to pay more to have a hedging position against future. Thus a spike in put option prices is expected.

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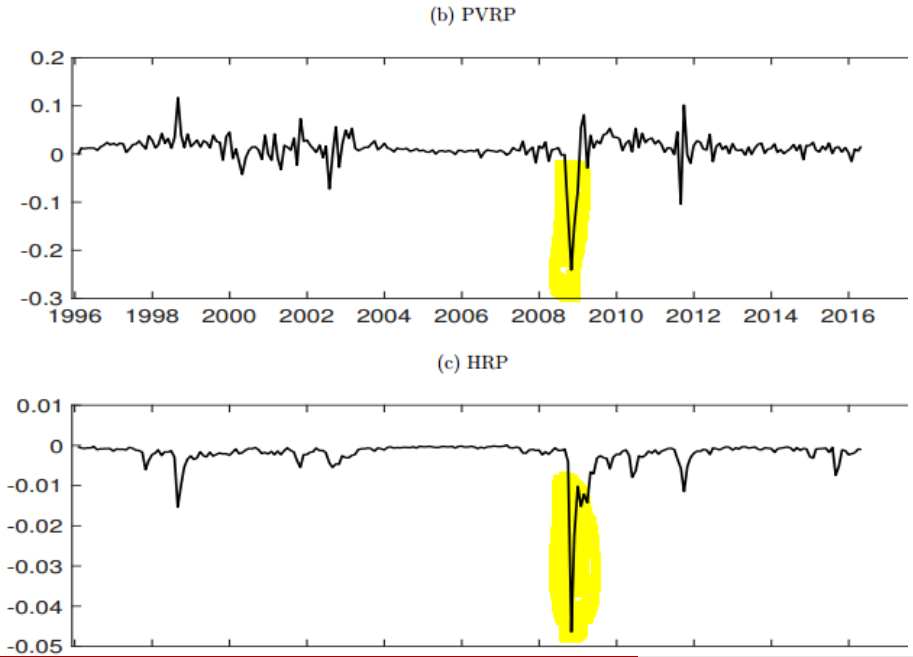
- But both negative spikes during the same time doesn’t make sense. Again, there is a strong mismeasurement concern.



Comment 2: Asset allocation exercise

- $w_t = \frac{\hat{r_{t+h}}}{\gamma \hat{\sigma^2_{t+h}}}$,
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  - $\Rightarrow$ I suspect that the w_t estimated in the paper might be very volatile because in theory the risk-neutral variance should reflects more than expected uncertainty but also some time variation in the risk aversion of the investor (Bekaert, Engstrom, and Xu (2019))

Conclusion

- I highly recommend it!
- To make it more convincing:
  1. Interpretations of negative spikes in the RP measures should be addressed in the paper
  2. The “economic interpretation exercises” should be further explained — what are the motivations, and what are the key messages? I also think the asset allocation exercise is a little unnecessary, but could be very interesting if done correctly

Thank You!

nancy.xu@bc.edu